PHY 114 SOLUTION SET 5

24.55. The removal of the dielectric results in change in the energy of the capacitor and the change in charge.

$\Delta U = U_f - U_i = \frac{1}{2} C_0 V^2 - \frac{1}{2} K C_0 V^2 = -\frac{1}{2} (K-1) C_0 V^2$

The work done to maintain a constant voltage is the voltage multiplied by the change in charge ~~due to battery~~.

$W_{bat} = V(Q_f - Q_i) = V(C_0 V - K C_0 V) = -(K-1) C_0 V^2$

The work done to remove dielectric is the difference between the change in capacitor's energy and work done by the battery.

$W = \Delta U - W_{bat} = -\frac{1}{2}(K-1) C_0 V^2 + (K-1) C_0 V^2$

$= \frac{1}{2}(K-1) C_0 V^2 = \frac{1}{2}(3.4-1)(8.8 \times 10^{-9} F)(100 V)^2 = 1.1 \times 10^{-4} J$

24.58. On inserting the mica, the capacitance of the system changes. The voltage is not changed as the same battery is used.

$\Delta Q = Q_{final} - Q_{initial} = (C_{final} - C_{initial}) V = (K C_{in} - C_{in}) V$

$= (K-1) C_{in} V = (7-1)(3.5 \times 10^{-9} F)(32 V) = 6.7 \times 10^{-7} C$

24.59. The capacitors are in parallel as the potential is same for both. Thus we find net capacitance keeping in mind that the area is changed by a factor of half from the original ~~area~~ area.

$C = C_1 + C_2 = K_1 \epsilon_0 \frac{\frac{1}{2} A}{d} + K_2 \epsilon_0 \frac{\frac{1}{2} A}{d}$

$= \frac{1}{2} \frac{\epsilon_0 A}{d} (K_1 + K_2)$

25.1. $I = \frac{\Delta Q}{\Delta t}$

charge of an electron is 1.6×10^{-19} C.

Charge flowing in 1 sec = $1.3 \times 1 = 1.3$ C.

No. of electrons carrying this charge = $\frac{1.3}{1.6 \times 10^{-19}} = 8.13 \times 10^{18}$ ~~electrons~~

25.6. a) Using Ohm's law - $R = \frac{V}{I} = \frac{240 V}{9.5 A} = 12.63 \Omega$

b) $\Delta Q = I \Delta t = (9.5A) \times 15$ mins $= 9.5A \times 15 \times 60$ secs. $= 8600$ C

25.17. Resistance is proportional to the length of a wire. If the resistance one part is four times that of other their length of the long part should be four times that of short part.

$l = l_{short} + l_{long} = l_{short} + 4\,l_{short} = 5\,l_{short}$

$\therefore\ l_{short} = \frac{1}{5} l = 0.2\,l, \quad l_{long} = l - l_{short} = 0.8\,l$

If $10\,\Omega$ is the resistance in the whole wire, resistance on short half is

$= 0.2 \times 10 = 2\,\Omega.$

Resistance in other part $= (10-2) = 8\,\Omega$ (as they are in series)

25.20. Voltage drop using Ohm's law -

$V = IR.$ $\qquad A = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

where $R = \frac{\rho l}{A}$

$\therefore\ V = \frac{I\rho l}{A} = \frac{I\rho l}{\frac{\pi d^2}{4}} = \frac{4 I \rho l}{\pi d^2} = \frac{4(12)(1.68\times10^{-8})(26)}{\pi (1.628\times10^{-3})^2} = 2.5\,V$

(using resistivity of copper here)

25.25. If the wire is cut the new wire has length half of original. The volume of the wire is constant.

$V_1 = \pi r_1^2 l_1 \qquad V_2 = \pi r_2^2 l_2 \qquad l_2 = \frac{l_1}{2}$

$V_1 = V_2$

$\pi r_1^2 l_1 = \pi r_2^2 l_2 = \pi r_2^2 \frac{l_1}{2}$

$\therefore\ r_2^2 = 2 r_1^2 \qquad \therefore\ A_2 = 2 A_1$

$R_1 = \frac{\rho l_1}{A_1} \qquad R_2 = \frac{\rho l_2}{A_2} = \rho\left(\frac{l_1}{2}\right) \times \frac{1}{2 A_1} = \frac{\rho l_1}{4 A_1} = \frac{R_1}{4}$

25.38. We know that $P = \frac{V^2}{R}$. Since the resistance of the bulb remains the same. $(R_1 = R_2)$

$\frac{P_1}{P_2} = \frac{V_1^2}{R_1} \times \frac{R_2}{V_2^2} = \frac{V_1^2}{V_2^2} = \left(\frac{120}{240}\right)^2 = \frac{1}{4}$

$P_{USA} = \frac{1}{4} P_{europe}$

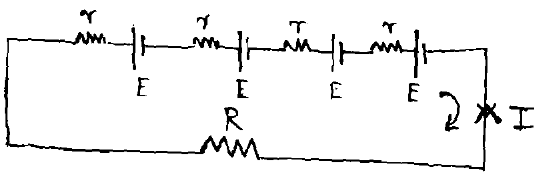
25.40. To find out amount of cost we need to find the work done times cost. $W = Pt$

$$\text{cost} = (25\,W)(365\,\text{day})\left(\frac{24\,hr}{1\,day}\right)\left(\frac{\$\,0.095}{kW\,hr}\right) \simeq \$\,21$$

25.60. The electric is related to potential as -

$$|\vec{E}| = \frac{\Delta V}{\Delta x} = \frac{70\times10^{-3}\,V}{10^{-8}\,m} = 7\times10^{6}\ V/m$$

26. 2.



The four cells are shown in the diagram. We now consider the potential over the whole circuit.

$$(E - Ir) + (E - Ir) + (E - Ir) + (E - Ir) - IR = 0$$

$$4(E - Ir) = IR$$

$$4Ir = 4E - IR$$

$$\therefore r = \frac{4E}{4I} - \frac{R}{4} = \frac{1.5}{0.45} - \frac{12}{4} = 0.33\,\Omega$$

26.16. a) 680 Ω and 820 Ω are in parallel which is in series with 960 Ω.

$$\frac{1}{R_1} = \frac{1}{680} + \frac{1}{820} \qquad \therefore R_1 = \frac{820\times680}{820+680} = 372\,\Omega$$

$$R_{net} = R_1 + R_2 = 372 + 960 = 1332\,\Omega$$

b) We now find current in circuit.

$$V = I R_{net}$$

$$\therefore I = \frac{12}{1332} = 9.009\times10^{-3}\,A$$

To find voltage across 960 Ω, $V_{960} = I \times 960 = 8.649\,V$

Since the total voltage in circuit is 12 V, voltage drop across other two resistors will be same (since they are parallel) and equal to $(12 - 8.649)\,V = 3.4\,V$

26.17. We find the resistance in each bulb using-

$$P = \frac{V^2}{R}$$

$$\therefore R_1 = \frac{V_1^2}{P_1} = \frac{(110)^2}{75} \qquad R_2 = \frac{V_2^2}{P_2} = \frac{(110)^2}{25}$$

Since resistors are in parallel, the net resistance is -

$$R_{net} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(110)^2}{75} \times \frac{(110)^2}{25} \times \frac{1}{\left(\frac{(110)^2}{25} + \frac{(110)^2}{75}\right)}$$

$$= \frac{(110)^2}{75 \times 25} \times \frac{25 \times 75}{(25+75)} = \cancel{\frac{HX14}{100}} = \frac{110 \times 110}{100}$$

$$= 121 \Omega$$

26.25. a) On closing the switch we added an additional resistor in parallel to the circuit reducing the resistance across the parallel portion of the circuit. Thus net resistance is lowered as well. More current will be delivered by the same battery due to lower resistance. This would increase voltage across R_1. Net voltage in circuit is same. This implies that voltage across R_3 and R_4 decrease which are in parallel.

V_1 increases along with V_2 which was zero.

V_3, V_4 decrease.

b) Using Ohm's law $V = IR$. For fixed resistance, ~~volt~~ change in circuit will be same as change in voltage. Thus I_1, I_2 increase with decrease in I_3, I_4.

c) We know that $P = IV$. Since both I, V increase the power supplied by circuit increases.

d) Before switch is ~~closed~~ - open -

R_3, R_4 in parallel and then in series with R_1.

$$R_{net} = R_1 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 125 + \left(\frac{2}{125}\right)^{-1} = 187.5 \Omega$$

$$I = \frac{V_{bat.}}{R_{net}} = \frac{22}{187.5} = 0.1173 \text{ A}$$

current through R_1 is 0.1173 A.

$$V_1 = I R_1 = (0.1173)(125) = 14.66 \text{ V}$$

$$V_1 + V_3 = V \qquad V_3 = V_4 \ (\because \text{in parallel})$$

$$V_3 = 22 - 14.66 = 7.34 \text{ V}$$

$$V_3 = I_3 R_3 \quad \therefore I_3 = \frac{7.34}{125} = \cancel{0.587 A} \quad 0.0587 \text{A}$$

Since $R_3 = R_4$. $\quad I_4 = I_3 = 0.\cancel{0}587\,A$.

When switch is closed –

R_2, R_3, R_4 are in parallel and then in series with R_1.

$$R_{net} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1}$$

$$= 125 + \left(\frac{3}{125}\right)^{-1} = 166.7\,\Omega$$

$$I = \frac{V}{R_{net}} = \frac{22}{166.7} = 0.1320\,A$$

$$V_1 = I_1 R_1 \quad \because I_1 = I$$

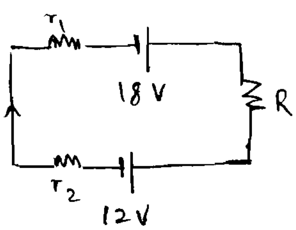
$$\therefore V_1 = 0.132 \times 125 = 16.5\,V$$

$V_1 + V_2 = V$ $\quad$ ($V_2 = V_3 = V_4$ all are in parallel connection)

$$V_2 = 22 - 16.5 = 5.5\,V$$

$$I_2 = I_3 = I_4 = \frac{5.5}{125} = 0.044\,A$$ ($\because R_2 = R_3 = R_4$, the current in them is same as well)

26.28.



Applying Kirchhoff's law to the circuit to find the current I, (note the orientation of the batteries are different along the loop)

$$-I(2) + 18 - I(6.6) - 12 - I(1) = 0$$

$$-3I - 6.6I = -6$$

$$\therefore I = \frac{6}{9.6} = 0.625\,A$$

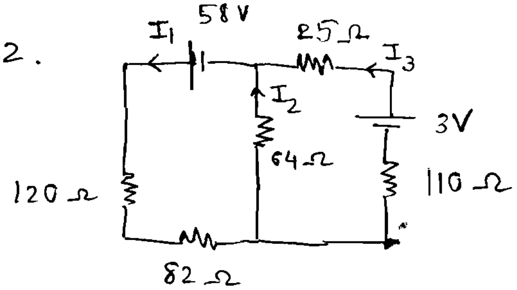
The terminal voltage is found by adding the voltage drop across internal resistance and the E.M.F. from left to right.

18V battery : $V = -I(2) + 18 = -(0.625A)(2\Omega) + 18\,V = 16.75\,V$

12V " : $V = I(1) + 12 = (0.625A)(1\Omega) + 12 = 12.625\,V$

The direction of current through the batteries is opposite for the two.

26.32.



We consider three currents I_1, I_2, I_3 flowing in the various loops but from conservation we see that.

$$I_1 = I_2 + I_3$$

Applying Kirchhoff's law to left loop we see-

$$58 - I_1(120) - I_1(82) - I_2(64) = 0$$

$$58 = \overset{202}{\cancel{200}}\, I_1 + 64 I_2 \qquad -①$$

Applying Kirchhoff's law to right loop we see-

$$3 - I_3(25) + I_2(64) - I_3(110) = 0$$

$$3 = 135 I_3 - 64 I_2 \qquad -②$$

Using $I_1 = I_2 + I_3$.

$$3 = 135(I_1 - I_2) - 64 I_2 = 135 I_1 \overset{-199}{\cancel{+71}} I_2 \qquad -③$$

$$\therefore\ 3 + 199 I_2 = 135 I_1 \qquad I_1 = \frac{(3+199 I_2)}{135}$$

Using ①,

$$58 = \overset{202}{\cancel{200}}\, I_1 + 64 I_2$$

$$= 202\left(\frac{3+199 I_2}{135}\right) + 64 I_2 = \cancel{4.88} + \cancel{188}\,\cancel{324.29 I_2} + 64 I_2$$

$$\cancel{388} - 4.\cancel{88} = \cancel{388}.29 I_2 \quad \therefore \cancel{I_2}$$

$$58 = 4.48 + (297.7 + 64) I_2$$

$$361.76 I_2 = 53.52 \qquad \therefore I_2 = 0.147 \text{ A}$$

$$I_1 = \frac{3 + 199 I_2}{135} = 0.24 \text{ A}$$

$$I_3 = I_1 - I_2 = 0.0919 \text{ A}$$

The current in the resistors are -

120 Ω, 82 Ω } 0.24 A

64 Ω } 0.15 A

25 Ω, 110 Ω } 0.092 A